Two-Dimensional Behavior of a Thin Web on a Roller – Part 2

By

Jerry Brown

Essex Systems

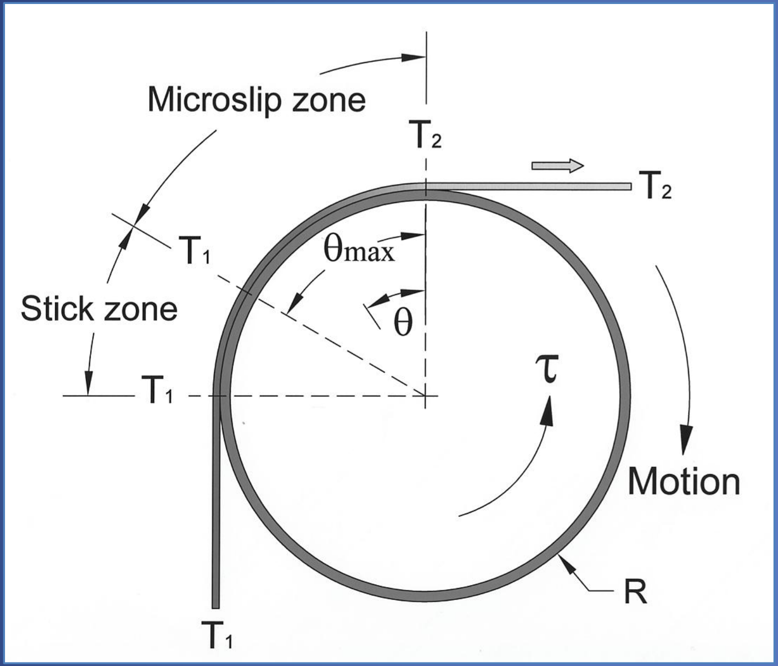
Four main topics will be covered

- A simple experiment that sheds light on how the stick and microslip zones work.
- A discussion of the role of lateral shear stress in microslip.
- A review of the entry slip criterion presented in a 2009 IWEB paper and its relationship to lateral shear effects.
- An improved nonlinear model for the web-on-roller geometry, using curvilinear coordinates.

Terminology - Microslip - (in a steady state)

- If torque is applied to a roller, either to brake or drive it, then tension in the web has to change between the entry and the exit. This means that the web must also stretch and slip on the rigid roller surface.
- This slipping is called microslip to distinguish it from the slipping that occurs when the web breaks completely free of the roller.
- Frictional forces that result from the microslip establish equilibrium between the tension change in the web and the applied torque.
- The direction of the microslip, relative to the roller surface, can be either in the direction of web motion or against it, depending on whether the tension is increasing or decreasing as the web moves through the microslip zone.

The capstan equation



$$\frac{dT}{d\theta} = \varphi T$$

φ is the coefficient of friction

$$\theta_{max} = \frac{1}{\varphi}\ln\left(\frac{T_1}{T_2}\right)$$

$$T = T_2 e^{\varphi\theta}$$

In this paper, θ is taken as negative CCW.

I chose to associate, a sign with φ, positive if $T_2 > T_1$ and negative otherwise. That takes care of the bookkeeping. But, if I could redo the paper, I would change that; because it takes attention away from the root cause, which is the direction of motion of the web relative to the roller. And that's exactly the kind of fuzzy thinking I wanted to eliminate.

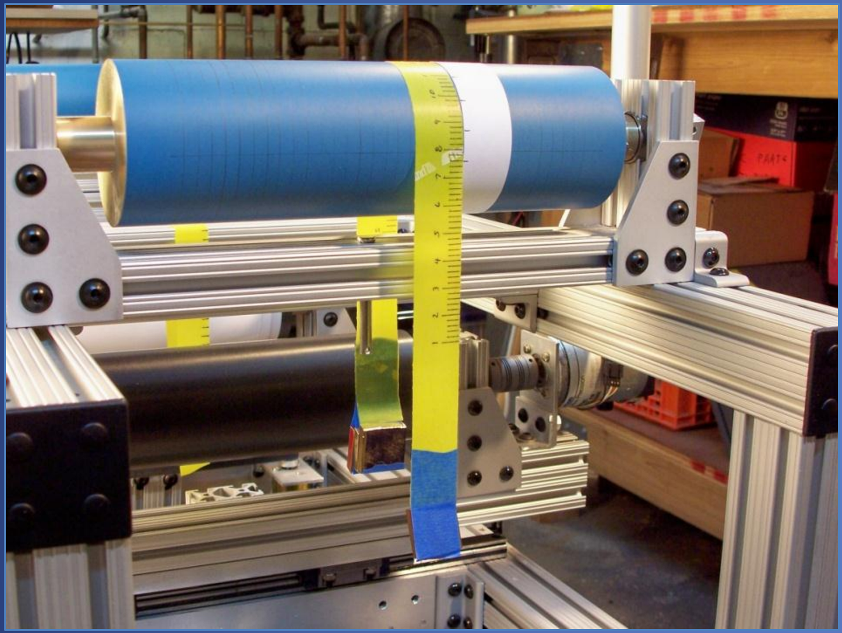
A closer look at the capstan model

- Caution: The following discussion applies only to a web that is in a semi-static situation. In a continuously moving web, with tension changes at the roller entry, things are more complicated.
- However, this doesn’t alter the main point, which is that much of a moving web’s behavior on a roller originates at the exit and propagates toward the entry.

Why worry about stick zone location?

- Because a better understanding of this question is critical to developing a full 2D model and it's not a trivial question.
- In earlier work, investigators have gone to considerable pains to explain the nature of the microslip zone.
  - Brandenburg in 1972 and Dwivedula in 2005 both used a discrete model consisting of a chain of solid blocks and springs wrapped onto a roller surface to explain how strain at the exit propagates up the chain
  - And they both demonstrated that making the elements progressively smaller leads to the capstan equation.
- When it comes to location, though, Brandenburg doesn't make an argument for location at all and Dwivedula used a thermodynamic argument to prove that it can only be at the entry.

The semi-static experiment



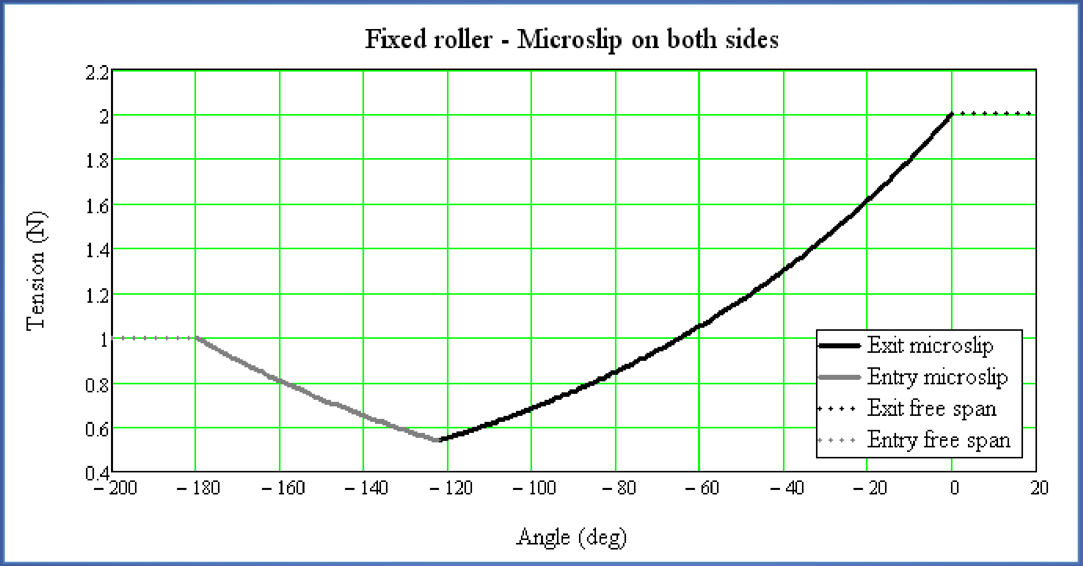
This is the experimental setup which will be the basis for discussion.

The belt is 1 inch wide and made of 0.005 inch (0.127 mm) latex. Test weights were 0.231 Lb (1.027 N) and 0.359 Lb (1.699 N). The coefficient of friction was 0.62.

The roller could be locked or released and turned by hand.

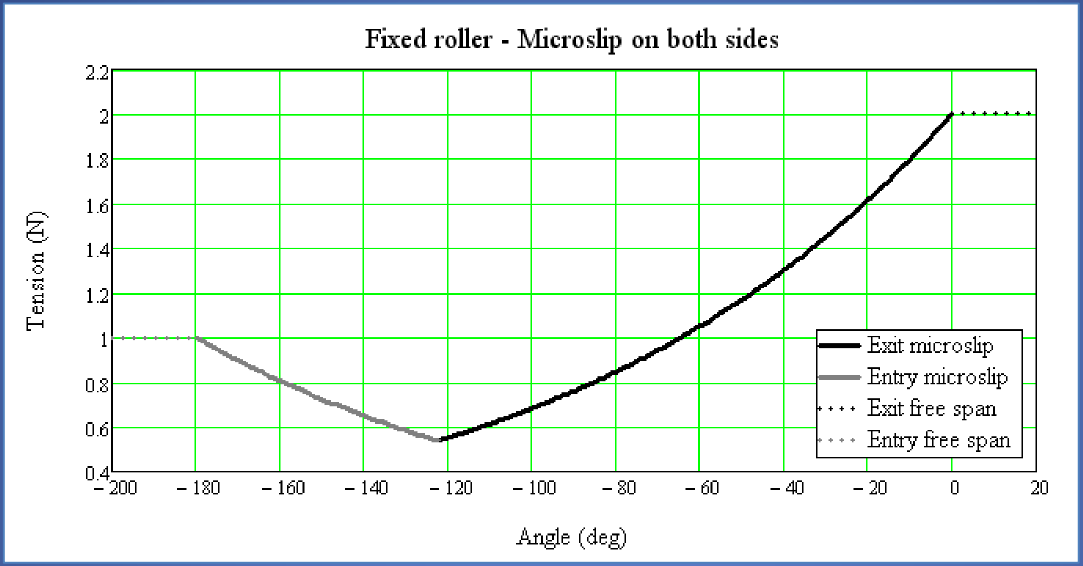
The scale on the latex was made with a ball point pen and metal scale. The maximum strain was 50%. A paper reference scale was taped to the roller. This was primarily used to get a qualitative sense of what was happening. Expectations for quantitative measurements were low because of the large strain and stiction. However, some data was taken and results will be presented after the theoretical discussion.

The traction graph



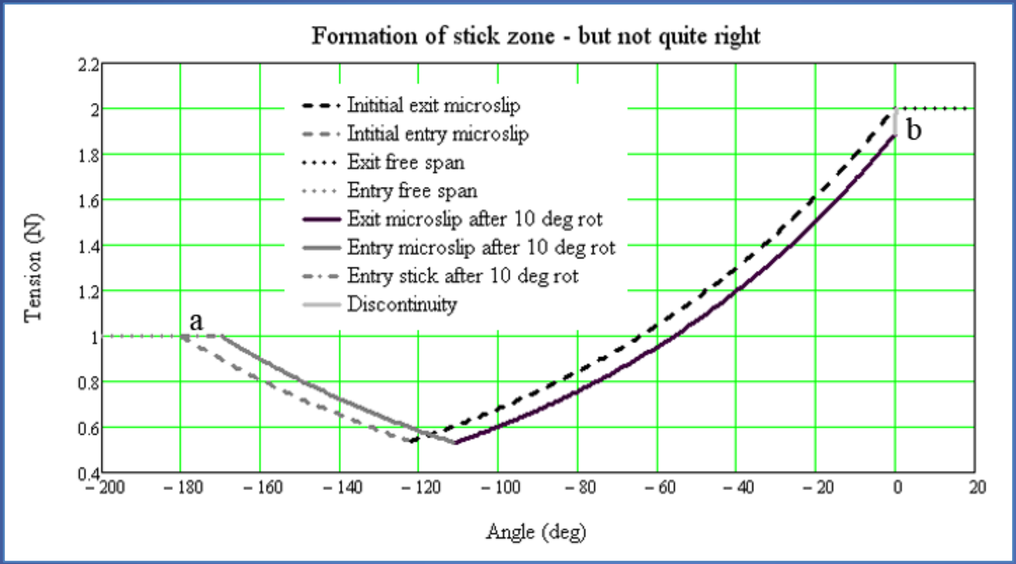
This and the following graphs are hypothetical and based on the capstan equation. To simplify the discussion, the weight at the exit end is assumed to be 2 N. At the entry it's 1 N. Under these circumstances, two microslip zones develop and they intersect at a point where the tensions on both sides simultaneously satisfy the capstan equation . On a fixed roller, like the one shown in the photo, microslip zones will be *active* only while they're forming. So, the web will quickly settle into a state of equilibrium and the microslip will become *inactive*.

The traction graph



It should be mentioned that this isn't the only possible microslip geometry. If the difference in the weights is reduced, a point will be reached where a constant tension zone develops between the two curves. If the difference in the weights is increased, a point is reached where one of the microslip zones consumes all of the wrap angle and the belt will slip off the roller.

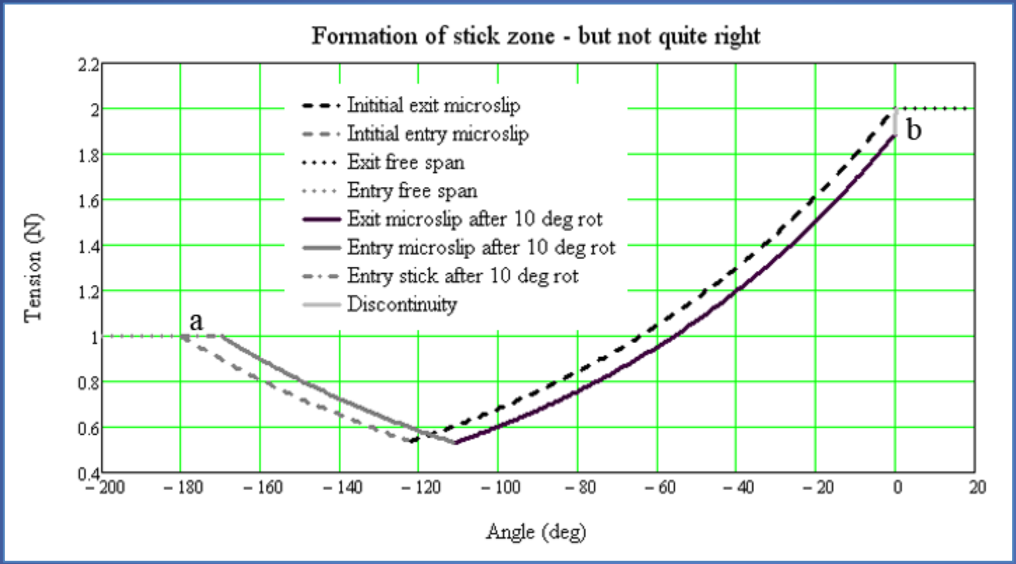
Now add a 10 degrees of rotation



$$\frac{dT}{d\theta} = \varphi T$$

Now, imagine that the roller is unlocked and slowly rotated by hand, 10 degrees toward the exit. An initial assumption might be that both zones move with the roller as shown above. At point (a), there is no problem with this assumption. The web enters onto the roller at the same tension it had in the free span. So, in segment (a), the derivative of T with respect to θ is zero and that's allowed by the capstan differential equation, because there is no relative motion between the web and roller to enable friction.

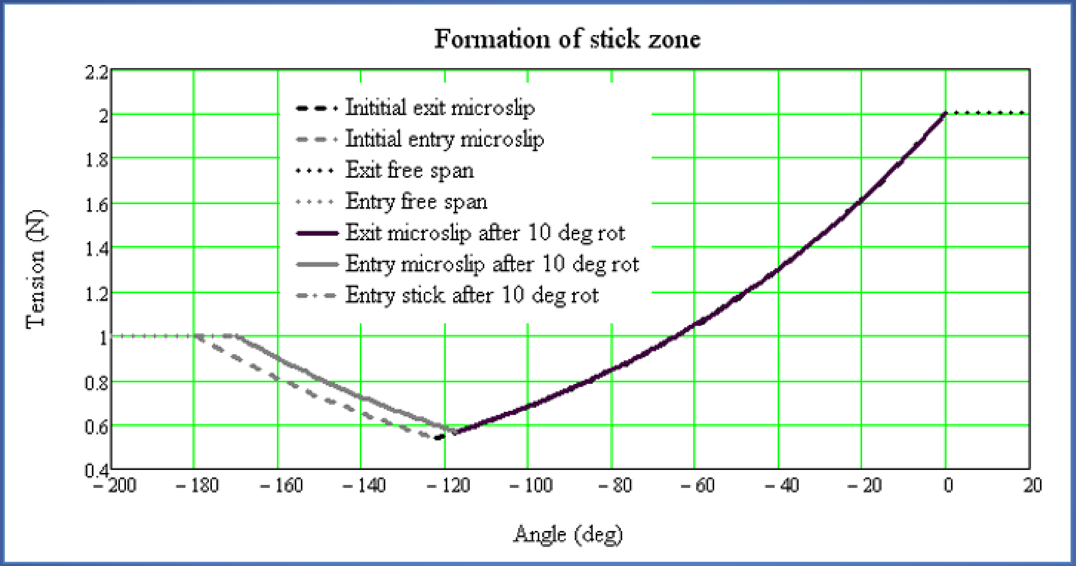
The situation at the exit



$$\frac{dT}{d\theta} = \varphi T$$

But, there is a problem at point (b). As the web moves off the roller into the free span, it will have to make a rapid change in tension. This would require the derivative of T with respect to θ to become very large – larger than the limit of imposed by friction in the differential equation . So, something else must happen there.

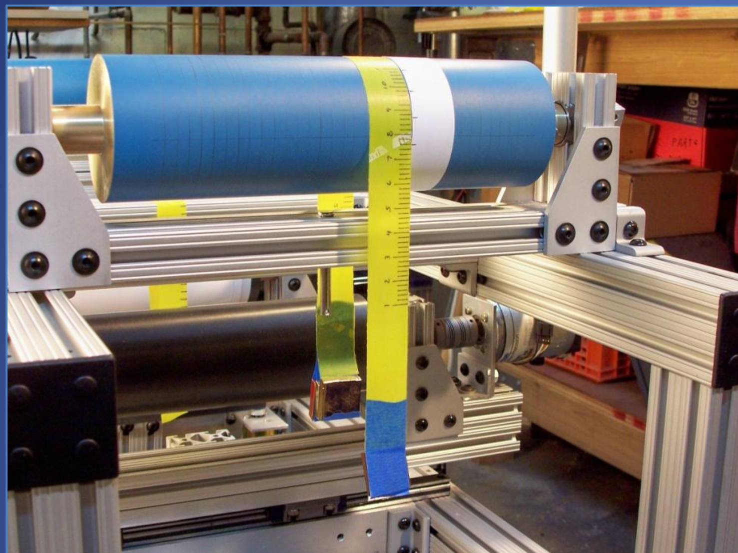
What really happens at the exit



$$\frac{dT}{d\theta} = \varphi T$$

As the roller begins rotating, the bit of web adjacent to the exit sees an increase in $dT/d\theta$. This goes up until φT is exceeded, causing that bit to slide into a new position on the roller. This creates a disturbance that travels upstream toward the entry at the speed of sound, causing each point, in turn, to be displaced in the same manner . This process stops at the point where the new exit segment meets the displaced entry segment. As the roller continues to rotate, this continues until all that's left is a constant tension stick zone at the entry and a microslip zone at the exit.

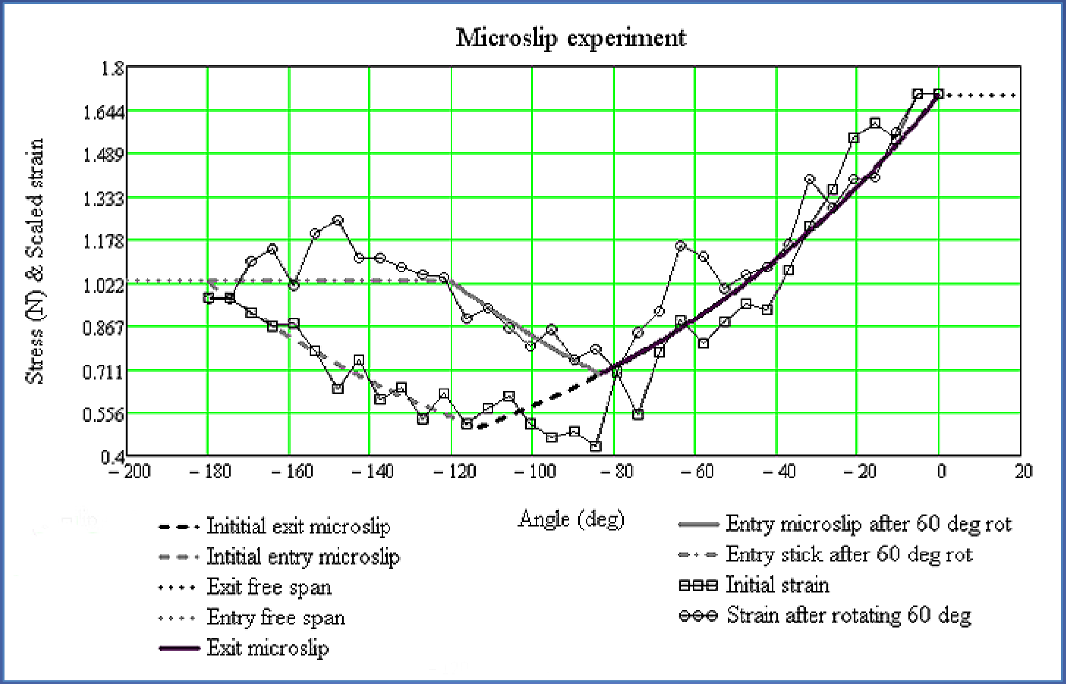
An experiment



The scale marks on the latex were 1/10 inch (2.54 mm) apart. Since they were made by hand using a metal scale, it was calibrated while relaxed and flat on a table, using a precision scale and a magnifying glass.

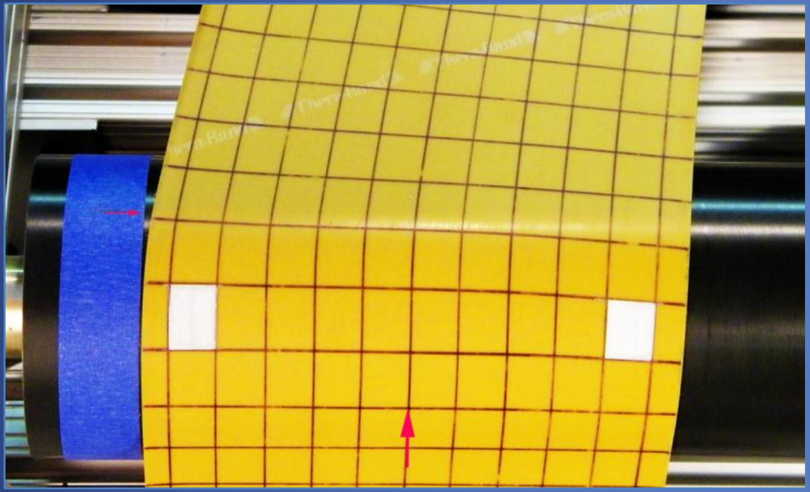
The strain was measured by taping a paper strip next to the latex scale and placing a pencil mark on the paper opposite each scale marking. This was the second largest source of error – on the order of 0.02 inch (0.5 mm). Then, the paper strip was taken off the roller, laid flat, and measured in the same way as the latex scale. Stick-slip was the biggest source of error. Efforts were made to help that with some cautious banging on the machine frame. The modulus of the latex wasn't independently measured. Instead, the strain data was scaled to make the value at the exit end equal to the numerical value of the test weight.

Data



It's apparent that a considerable amount of stick-slip remained. The data shown here was smoothed using a running average with a coefficient of 0.5. Significant Poisson contraction was evident and may also have affected the assumption of uniaxial stress. However, everything considered, the data fits surprisingly well. It's good enough to provide at least modest assurance that the reasoning is correct.

Microslip on idlers



Microslip can be present without any net transfer of torque to a roller.

This shows a latex web on the roller upstream of a misaligned roller. It's well known that the web deflection creates a moment there and the grid shows the MD strain differential. On the low tension side it extends almost to the entry.

Shear microslip is also evident in the gap between the left edge and the tape. The red arrow on the tape marks the exit line.

It's evident that the microslip could interfere with the entry conditions long before the tension distribution is bad enough to cause the web edge to lift.

Something like this could be happening with a cambered web.

The entry slip criterion

- I see no reason to doubt the entry slip criterion derived in the 2009 Part 1 paper.
- It's a necessary condition for existence of a stick zone.
- However, by itself, it isn't sufficient because it takes no account of what happens in the rest of the wrap.
- A concise derivation is in the paper. The net result is,

$$\sqrt{\left(\frac{\partial \sigma_{xy}}{\partial x}\right)^2_{at\,entry} + \left(\frac{\partial \sigma_{xx}}{\partial x}\right)^2_{at\,entry}} \leq \varphi \frac{|\sigma_{xx}|}{R}$$

Incorrect in the paper.

Note: The *y*-derivatives were used in the 2009 paper. But, since the stresses in this equation exist in the free span at the entry,

$$\left(\frac{\partial \sigma_{xy}}{\partial x}\right)^2 = \left(\frac{\partial \sigma_{yy}}{\partial y}\right)^2 \quad and \quad \left(\frac{\partial \sigma_{xx}}{\partial y}\right)^2 = \left(\frac{\partial \sigma_{xy}}{\partial y}\right)^2$$

A rearrangement of the capstan equation

In the Part 1 paper, it was shown that both sides of the equation we began with could be multiplied by the reciprocal of the roller radius 1/R and that Rθ could then be replaced by a Cartesian coordinate such as *x*. And if the tension *T* is changed to stress σ_{xx} we get something that looks like a one-dimensional equilibrium equation, where it's understood that the right hand side is the result of a stress that's pressing the web against a rigid surface.

$$\frac{d\sigma_{xx}}{dx} = \varphi \frac{|\sigma_{xx}|}{R}$$

With this in mind and considering the earlier discussion about microslip behavior it's not entirely unreasonable to consider a capstan equation for shear. Although, this requires a number of questionable assumptions, it could be instructive.

A capstan equation for shear

The first assumption is that in a shear stress microslip zone, MD stress is constant. This can't be strictly true. But, MD stress is generally much larger than the shear stress. So, it may not be totally irresponsible. To the extent that it's wrong, we have to also question the validity of the MD capstan equation.

The next assumption is that shear stress should face the same situation as MD stress, in regard to the location and development of the microslip zone. This is completely plausible.

With those assumptions, a capstan equation for shear would look like this, where σ_{xx} is assumed to be constant.

$$\frac{d\sigma_{xy}}{dx} = \varphi \frac{|\sigma_{xx}|}{R}$$

Results of integration

Shear stress usually has a non-uniform profile. At a misaligned roller, for example, the distribution is parabolic. But, for the moment, we will assume that that it has a uniform, width-wise distribution with the same average value as the non-uniform profile.

The equation is even easier to integrate than its companion. If σ_{xy1} is the entrance value and σ_{xy2} the exit value, x is taken as negative for CCW θ and the right hand term is positive when $\sigma_{xy2} > \sigma_{xy1}$, the results are,

$$\sigma_{xy} = \varphi \frac{|\sigma_{xx}|}{R} x + \sigma_{xy2} \quad or \quad F_{xy} = \varphi F_x \theta + F_{xy2}$$

$$\theta_{\max} R = \left(\sigma_{xy1} - \sigma_{xy2}\right) \frac{R}{\varphi |\sigma_{xx}|} \quad or \quad \theta_{\max} = \left(F_{xy1} - F_{xy2}\right) \frac{1}{\varphi |F_x|}$$

F_{xy} represents the lateral force (or force/width) and F_x represents the tension in units of force (or force/width).

Comparison with current practice

Looking at the last equation from another point of view, we can write,

$$F_{xy} = \varphi \theta_{\max} F_x$$

Wrong in paper

Where F_{xy} is the maximum lateral force difference that can be supported across the roller, F_x is the MD tension in units of force and θ_{max} is the total wrap angle. This is the same result produced by a straightforward static calculation without regard to microslip and is in agreement with current practice - at least by Tim Walker.

The significant thing about looking at this from the standpoint of microslip is that the shear stress profile is really parabolic and could be producing traction problems in the center part of the web well before reaching the limit of the static criterion.

A sample calculation

To get a little feeling for how much shear matters, values were calculated for a misaligned roller using an FEA model.

Length	60 inches (1.52 m)
Width	40 inches (1.02 m)
Thickness	0.001 inch (25.4 microns)
Tension	0.5 pli [500 psi (3.5 MPa)]
Modulus	50,000 psi (0.34 GPa)
Poisson ratio	0.35
Roller radius	3 inches (76 mm)
Roller Angle	1 deg (0.017 radian)
Coefficient of friction	0.25

← Missing in paper

The average shear stress is 53.4 psi (0.37 MPa). So, θ_{max} is 24.5 degrees (0.43 radian)

It's worth noting that the FEA analysis indicated that the entry slip criterion, defined earlier, was barely met.

The 2D+*w* model for a web on a roller
(in cylindrical curvilinear coordinates)

For a detailed discussion of the 2D+*w* model, please refer to the paper presented earlier.

For a cylinder with *y*-axis symmetry, radius *r*, and azimuth angle θ, the coordinate transformations are,

$$x = r\cos(\theta) \qquad y = y \qquad z = r\sin(\theta)$$

The deformed curvilinear coordinates for the stressed web will be denoted using tildes .

The displacement variables in the cylindrical coordinate system will be *u* for the θ-direction, *v* for the *y*-direction and *w* for the *r*-direction.

Although r will be eliminated as a variable in this problem it still has role as a constant. To make that fact easier to remember, *r* will be capitalized.

Notation

The following definitions not only make subsequent equations more compact. They also group terms that have special relevance in nonlinear elasticity as components of expressions for strain and the angles between relaxed and deformed coordinates.
In linear theory, they actually become the strains, shears and rotations.

$$e_{\theta\theta} = \frac{1}{R}\frac{\partial u}{\partial \theta} \qquad e_{yy} = \frac{\partial v}{\partial y}$$

$$e_{\theta y} = \frac{1}{R}\frac{\partial v}{\partial \theta} + \frac{\partial u}{\partial y} \qquad e_{\theta r} = \frac{1}{R}\left(\frac{\partial w}{\partial \theta}\right) \qquad e_{yr} = \frac{\partial w}{\partial y}$$

$$\omega_{\theta} = \frac{1}{2}\frac{\partial w}{\partial y} \qquad \omega_{y} = \frac{1}{2}\frac{1}{R}\frac{\partial w}{\partial \theta} \qquad \omega_{r} = \frac{1}{2}\left(\frac{1}{R}\frac{\partial v}{\partial \theta} - \frac{\partial u}{\partial y}\right)$$

Equations of equilibrium

$$\frac{1}{R}\frac{\partial}{\partial\theta}\left[\sigma_{\tilde\theta\tilde\theta}\left(1+e_{\theta\theta}\right)+\sigma_{\tilde\theta\tilde y}\frac{\partial u}{\partial y}\right]+\frac{\partial}{\partial y}\left[\sigma_{\tilde y\tilde\theta}\left(1+e_{\theta\theta}\right)+\sigma_{\tilde y\tilde y}\frac{\partial u}{\partial y}\right]$$
$$+\frac{1}{R}\left(\sigma_{\tilde\theta\tilde\theta}\frac{1}{R}\frac{\partial w}{\partial\theta}+\sigma_{\tilde\theta\tilde y}\frac{\partial w}{\partial y}\right)=0$$

θ-direction

$$\frac{1}{R}\frac{\partial}{\partial\theta}\left[\sigma_{\tilde\theta\tilde\theta}\frac{1}{R}\frac{\partial v}{\partial\theta}+\sigma_{\tilde\theta\tilde y}\left(1+e_{yy}\right)\right]+\frac{\partial}{\partial y}\left[\sigma_{\tilde y\tilde y}\left(1+e_{yy}\right)+\sigma_{\tilde y\tilde\theta}\frac{1}{R}\frac{\partial v}{\partial\theta}\right]=0$$

y-direction

$$\frac{1}{R}\frac{\partial}{\partial\theta}\left(\sigma_{\tilde\theta\tilde\theta}\frac{1}{R}\frac{\partial w}{\partial\theta}+\sigma_{\tilde\theta\tilde y}\frac{\partial w}{\partial y}\right)+\frac{\partial}{\partial y}\left(\sigma_{\tilde y\tilde\theta}\frac{1}{R}\frac{\partial w}{\partial\theta}+\sigma_{\tilde y\tilde y}\frac{\partial w}{\partial y}\right)$$
$$-\frac{1}{R}\left[\sigma_{\tilde\theta\tilde\theta}\left(1+e_{\theta\theta}\right)+\sigma_{\tilde\theta\tilde y}\frac{\partial u}{\partial y}\right]=0$$

r-direction

If $w = 0$ these go away.

All derivatives and terms involving the *r*-coordinate have been removed, so that the equations will represent a membrane. In its relaxed state it is shaped like the surface of a cylinder (symmetrical about the *y*-axis), and *w* is any displacement from that surface caused by a load applied to it.

The third term

$$-\frac{1}{R}\left[\sigma_{\tilde{\theta}\tilde{\theta}}\left(1+e_{\theta\theta}\right)+\sigma_{\tilde{\theta}\tilde{y}}\frac{\partial u}{\partial y}\right]=0$$

r-direction

Here is what's left of the *r*-direction equation. Only the first two equations should be needed when there is no radial displacement. So, what's wrong? The trouble is due to the fact that the web is being pressed against a roller and the roller is providing a reaction pressure to support it. The mathematics obediently created the radial pressure of the web. However, nothing in the derivation of the equations presumed the existence of a reaction. So, the existence of the roller cancels out the last group. It's going to reappear, though, in the first two equations as part of friction terms.

It should also be noted that the quantity inside the brackets is identical to the term inside the first set of brackets of the θ-direction equation.

$$\frac{1}{R}\frac{\partial}{\partial\theta}\left[\sigma_{\tilde{\theta}\tilde{\theta}}\left(1+e_{\theta\theta}\right)+\sigma_{\tilde{\theta}\tilde{y}}\frac{\partial u}{\partial y}\right]+\frac{\partial}{\partial y}\left[\sigma_{\tilde{y}\tilde{\theta}}\left(1+e_{\theta\theta}\right)+\sigma_{\tilde{y}\tilde{y}}\frac{\partial u}{\partial y}\right]=0$$

θ-direction

Unrolling the web

Finally, all of the products $R\partial\theta$ can be replaced by a new variable which will be called *x*. Making this change and setting *w* equal to zero, the equilibrium equations for a web on a roller with friction are,

$$\frac{\partial}{\partial x}\left[\sigma_{\tilde{x}\tilde{x}}\left(1+e_{xx}\right)+\sigma_{\tilde{x}\tilde{y}}\frac{\partial u}{\partial y}\right]+\frac{\partial}{\partial y}\left[\sigma_{\tilde{y}\tilde{x}}\left(1+e_{xx}\right)+\sigma_{\tilde{y}\tilde{y}}\frac{\partial u}{\partial y}\right]=\Omega_x$$

$$\frac{\partial}{\partial x}\left[\sigma_{\tilde{x}\tilde{x}}\frac{\partial v}{\partial x}+\sigma_{\tilde{x}\tilde{y}}\left(1+e_{yy}\right)\right]+\frac{\partial}{\partial y}\left[\sigma_{\tilde{y}\tilde{y}}\left(1+e_{yy}\right)+\sigma_{\tilde{y}\tilde{x}}\frac{\partial v}{\partial x}\right]=\Omega_y$$

The terms Ω_x and Ω_y are friction functions.

Strains and stresses

These strains are aligned with the coordinates of the stressed web. But they are defined in term of the Cartesian system of the relaxed web (Green-Lagrange strains).

$$\varepsilon_{\tilde{x}\tilde{x}} = e_{xx} + \frac{1}{2}\left[{e_{xx}}^{2} + \left(\frac{\partial v}{\partial x} \right)^{2} \right] \quad \varepsilon_{\tilde{y}\tilde{y}} = e_{yy} + \frac{1}{2}\left[{e_{yy}}^{2} + \left(\frac{\partial u}{\partial y} \right)^{2} \right] \quad \varepsilon_{\tilde{x}\tilde{y}} = e_{xy} + e_{xx}\frac{\partial u}{\partial y} + e_{yy}\frac{\partial v}{\partial x}$$

These are the corresponding stress-strain relations – Hook's law again. The stresses are also aligned with the deformed coordinates of the stressed web (Cauchy stresses).

$$\sigma_{\tilde{x}\tilde{x}} = \frac{E}{1-\mu^{2}}\left(\varepsilon_{\tilde{x}\tilde{x}} + \mu\varepsilon_{\tilde{y}\tilde{y}} \right) \quad \sigma_{\tilde{y}\tilde{y}} = \frac{E}{1-\mu^{2}}\left(\varepsilon_{\tilde{y}\tilde{y}} + \mu\varepsilon_{\tilde{x}\tilde{x}} \right) \quad \sigma_{\tilde{x}\tilde{y}} = \frac{E}{2\left(1+\mu\right)}\left(\varepsilon_{\tilde{x}\tilde{y}} \right)$$

Accounting for friction

The problem now remains to determine the friction functions .

There will be components of friction in both the *x* and *y* directions, depending on conditions in the entry and exit spans. So, it is treated as a vector quantity.

In the microslip zone it will take its maximum value

$$\sqrt{\Omega_x{}^2 + \Omega_y{}^2} = \varphi \frac{1}{R} \left| \sigma_{\tilde{x}\tilde{x}} \left(1 + e_{xx}\right) + \sigma_{\tilde{x}\tilde{y}} \frac{\partial u}{\partial y} \right|$$

In the stick zone,

$$\sqrt{\Omega_x{}^2 + \Omega_y{}^2} < \varphi \frac{1}{R} \left| \sigma_{\tilde{x}\tilde{x}} \left(1 + e_{xx}\right) + \sigma_{\tilde{x}\tilde{y}} \frac{\partial u}{\partial y} \right|$$

Accounting for friction

Also in the stick zone, (following the same reasoning as in the earlier discussion of the entry slip criterion), the following relationships will hold.

$$\Omega_x = -\frac{\partial}{\partial x}\left(\sigma_{\tilde{x}\tilde{x}} \left(1 + e_{xx}\right) + \sigma_{\tilde{x}\tilde{y}} \frac{\partial u}{\partial y} \right)\Bigg|_{at\ entry}$$

$$and \quad \Omega_y = -\frac{\partial}{\partial x}\left(\sigma_{\tilde{x}\tilde{x}} \frac{\partial v}{\partial x} + \sigma_{\tilde{x}\tilde{y}} \left(1 + e_{yy}\right) \right)\Bigg|_{at\ entry}$$

The equations of the last few slides are not sufficient to create a working FEA model. Something more is needed to define the direction of microslip – probably the relative velocity between the web and the roller. But, this is getting closer.

The rest will have to wait for Part 3.

Conclusions

Location of the stick zone:

Changes in tension in the microslip zone propagate from the exit toward the entry and this explains why the stick zone is at the entry.

Hypothesis of a shear microslip zone:

Lateral stresses may propagate from back to front in the same manner as longitudinal stress and either contribute to microslip zones or create their own, even on an idler.

Because of the linear relationship between shear-microslip and wrap angle, the amount of wrap necessary to prevent slipping can be calculated using current methods. However there may often be situations in which the lateral profile of shear stress, which is usually non-uniform, causes it to penetrate into the line of entry causing loss of traction over only the central part of the web. This could aggravate wrinkling problems.

Conclusions

Progress toward a complete model:

Some additional progress over Part 1 has been made toward a full 2D model of traction on a roller. The 2D+*w* model with cylindrical curvilinear coordinates incorporates the features of nonlinear elasticity and the terms for radial pressure appear in the results as a natural consequence of a well-established formal procedure. And a better understanding of the nature of microslip has been gained through testing and analysis.

Time for Q&A

- There are quite a few typographical errors in the paper. If you would like a corrected copy, just send me a note essexsys.com.