Seeing the Invisible:
The Deformations and Stresses That Move Webs and The Two Rules That Govern Them.

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Introduction:

Web defects such as wrinkles and lateral misalignment are easy to see. However, the deformations and stresses that cause them are usually invisible to the unaided eye. This makes it very difficult for web handling engineers to relate cause and effect. As a consequence, there is still much controversy and confusion over issues such as,

1. Where do wrinkles come from and what can you do about them if they occur?
2. What causes a web to track laterally on a misaligned roller?
3. Should a concave roller be used for spreading instead of a bowed roller?
4. Does a concave roller even spread?
5. Should rollers be rough or slick to get rid of wrinkles?

The invisibility problem:

Stress is invisible. We experience it only through the deformations it produces. And in most cases, the deformations are so small that they can't be detected without special instruments such as strain gages, load cells and polarimeters. Even when deformations are visible, as in the case of the lateral deflection of a web at a misaligned roller, it is usually the deformations we *can't* see that lead to problems. For example, lateral deflection is often accompanied by shear strain. The shear strain is invisible. But, it can cause wrinkles.

The complexity problem:

To make matters worse, stress is mathematically difficult. It's not a scalar quantity like temperature. It's not even a vector like velocity. It's a squirrely thing called a tensor. And even people who understand tensor algebra will tell you that it's precious little help in understanding anything in an intuitive way. If you studied strength of materials or elasticity in school, your textbook probably avoided tensors with mind-bending graphic tools like Mohr's circle. So, even *thinking* about stress is hard.

Solving the invisibility problem:

The root of the invisibility problem is that webs are generally only stretched a few tenths of a percent. There are two reasons for this. First, webs need to be processed in a condition similar to the final application. For example, a web printed at an elongation of 10 % would look distorted when relaxed. Second, most plastics are permanently deformed at elongations of more than a few percent. Therefore, process lines typically run at tensions producing only a few tenths of a percent of elongation – that is .001 to .002 inch per inch.

In the present situation, where the goal is demonstration and insight, there is no reason why the elongation limitation can't be overcome by using a rubber web. Latex can be stretched several hundred percent without damage. It does exhibit some bad characteristics such as viscoelasticity and nonlinearity. But, then again, so do most of the plastics used for film production. By running at elongations of ten percent (0.1 inch per inch) it should be possible make deformations visible without provoking much more nonlinear behavior than is seen in ordinary webs. Printing a rectangular grid on the web can further enhance visibility of the deformation.

Another advantage of using a rubber web is that it has a high Poisson ratio, which amplifies the MD and CD interaction.

A special machine, illustrated in Figure 1, was built to facilitate testing with a latex web at high elongations.

Web: 6 mil latex, 6 inches wide, 20 feet long, imprinted with a reference grid.
Rollers: 10 inch face length, 3 inch diameter.
Drive: Closed-loop velocity control with tachometer feedback - 0 to 20 Ft/min
Tension control: Magnetic particle brake, no feedback - 0.5 to 10 Lb
Lateral position control: Closed loop control at unwind and rewind with edge sensors.
Rewind tension: Constant torque, no feedback – minimum possible.

During test runs, the working tension is isolated from the unwind and rewind to minimize winding issues. Winding tension is on the order of 0.5 Lb and a complete roll has less than 25 layers.

In a typical run, 10 spans of web pass through the 18 inch test span. At that point it will have reached 99.995 % of its steady state position. The web is then rewound by hand at low tension by threading it directly from the rewind to the unwind.

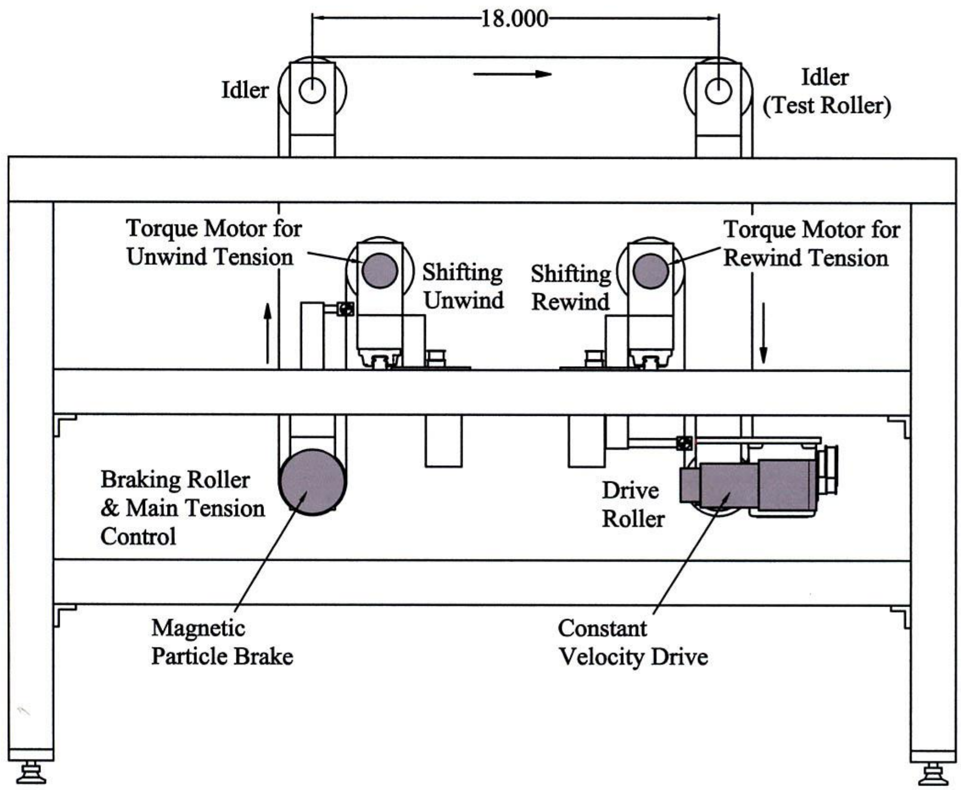

Figure 1
Test machine

Solving the complexity problem:

Part of the complexity problem is solved by solving the visibility problem. Seeing always aids understanding. But, the ultimate purpose of understanding is to find organizing principles that can be used to predict behavior when conditions are changed. There are two concepts that are crucial to understanding web/roller interaction.

The rules are:

The normal entry rule[1]

A web entering onto a roller will align its direction of travel perpendicular to the roller axis. If the web is not initially perpendicular, it will travel laterally on the roller at a rate proportional to the tangent of the angle between the web and the roller until it reaches the perpendicular condition.

The normal strain rule[2]

If a small patch of web is followed from the entry of one roller to the entry of the next, the ratio of the MD lengths of the patch will be the same as the ratio of the respective MD velocities at those points. Or in other words, $(1 + \varepsilon_u)/(1 + \varepsilon_d) = V_u/V_d$. Where ε_u is the MD strain at the entry of the upstream roller, ε_d is the strain at the entry of the downstream roller and V_u and V_d are the respective MD velocities

In this case MD is assumed to refer to the direction normal to the axis of a roller (even if the roller is misaligned) rather than being parallel to an imaginary line down the center of the process.

Notice that this rule implies that at the point of entry onto a roller, the MD strain can be predicted solely from web velocities at the rollers and the MD strain in the previous span. It does not depend on the CD or z components. Understanding this concept is one of the keys to developing an intuitive understanding of web/roller behavior.

In addition to providing intuitive insight, the normal strain rule also makes it possible, with help from the normal entry rule and elasticity theory, to use FEA software to solve for all of the stresses and strains throughout a span. This method was described by the author in a paper delivered at IWEB 2005[1]. Although that is not the subject of this paper, it is worth noting that a CAD tool with this capability will soon be available.

It is important to bear in mind that these rules apply only if there is no slipping between the web and the roller. Slipping may occur due to excessive friction in roller bearings. And at high speeds, air entrainment can reduce web to roller friction. However, this does not diminish the importance of the rules. Many processes operate at speeds where air flotation effects are not significant and rollers turn freely. And even when slipping does occur, it is useful to understand what is being lost.

The rest of this paper will use diagrams and pictures of an actual web running at large strains to accomplish the following.

1. Illustrate the basic principles underlying the two rules.
    a. Simplified string model for the normal entry rule.
    b. The normal entry rule in a 2-D web.

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[1] There is no evidence that any of the guiding companies in the U.S. knew of the normal entry rule prior to 1960. The first published mention of it is in a book by Donald Campbell published in 1958. The author independently rediscovered it in 1960 while working at Fife Corporation.

[2] Discovered by the author in February of 2003 and first published at IWEB 2005.

c. How conservation of mass and particle paths lead to the normal strain rule.

2. Show how the normal entry rule causes a uniform web, running between aligned rollers, to reach a state of pure MD strain.
3. Show how, in the case of a misaligned roller, a web path curves due to the 2-D effects predicted by Shelton's beam model.
4. Illustrate shear and rotation at a misaligned roller.
5. Illustrate how a concave roller spreads a web.
6. Show how operation of the normal strain rule at a nonuniform roller can cause the relaxed state of a web to become nonrectangular (which raises the question of how do you predict the deformations of a web under stress when you don't know its relaxed shape).
7. Show how strain is transported across a roller.
8. Show how a wrinkle evolves.

A note about roller traction:

When a web travels over a roller it will pass through two zones. The first is called the "stick" zone where there is no relative motion between the web and the roller. The second is called the "slip" zone where the web is beginning to slip loose under the influence of tension in the following span. As turning resistance of the roller increases or web-to-roller friction decreases, the slip zone will increase at the expense of the stick zone. When the stick zone goes to zero the surface speed of the roller will no longer match the web speed and neither the normal entry or normal strain rules will be operative. For a good discussion of this subject see a paper by Dilwyn Jones[2].

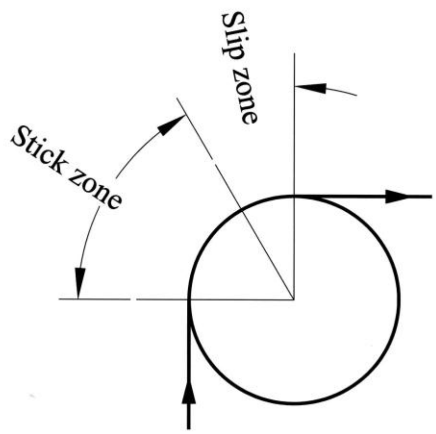

Figure 2
Roller traction

The reason for the Normal Entry Rule:

The simplest way to think about the normal entry rule is to imagine the web as a flexible string. This is illustrated in Figure 3.

Points on the web near the downstream roller, such as point (a) will move in the same direction as the roller surface. So, if the web is at an angle, θ, with the roller axis, those points will arrive at the roller at a point (a'), as though the web were following a helical path on the roller.

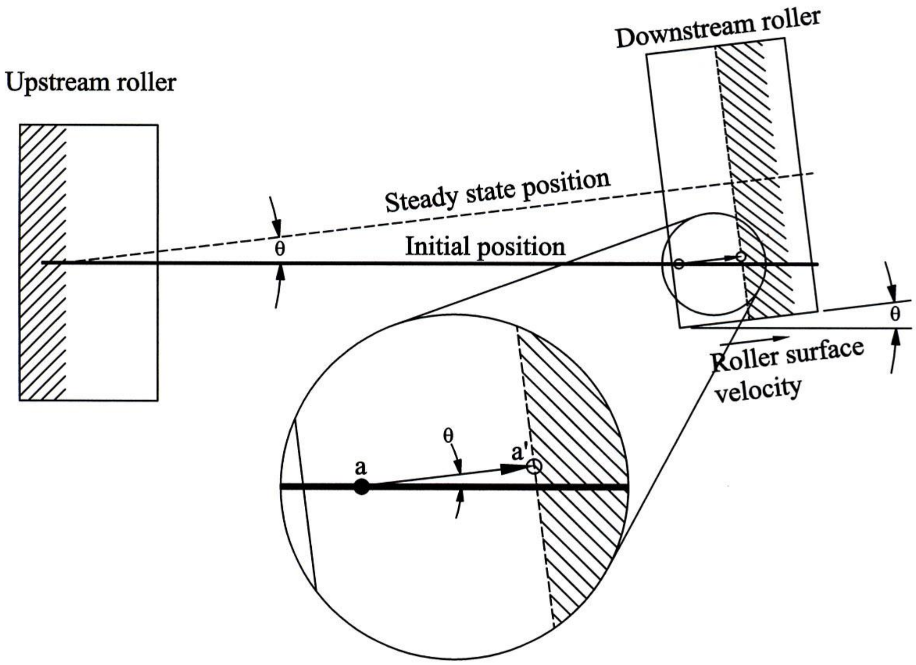

Figure 3
The normal entry rule

The web path is anchored at the upstream roller. So, as it advances in the MD direction it has no choice but to pivot in a manner that gradually reduces the angle θ. When it reaches the point where θ is zero it stops moving laterally.

You can easily see this by resting your fingers on a pencil and rolling it across a desktop. That's how I first saw it.

A real web is a little more complicated. But, the same principle is at work. The only difference is that the web can't take the idealized shape shown in Figure 3. Shelton[3] in 1968 showed that it can be treated as a deflected beam that follows a curved path between the rollers. The curve changes the effective pivot point for the web. But, the normal entry rule still applies at the downstream roller.

The reason for the Normal Strain Rule:

In Figure 4 below, the web is assumed to be running in a steady state with good traction on the rollers. At the entry to the rollers, the normal entry rule requires that the particle paths be normal to the roller axis. The area marked (a) represents a small area with sides parallel to two particle paths. The area marked (a') is the same portion of web at the moment that it enters onto the downstream roller. The roller surface velocities are V_u and V_d. Δx is the relaxed length of the area (a) in the MD direction. ε_{xu} is the MD strain at the upstream roller. ε_{xd} is the MD strain at the downstream roller. Conservation of mass requires that the mass flow into and out of the span for any portion of web between two particle paths be constant. Otherwise, material would accumulate in the span and a steady state would not exist. For this to be true, each area must travel past the line of roller contact in the same length of time, Δt. Therefore,

$$\Delta x\left(1+\varepsilon_{xu}\right)=V_u\,\Delta t \quad (1)$$

and

$$\Delta x\left(1+\varepsilon_{xd}\right)=V_d\,\Delta t \quad (2)$$

Equating Δt from (1) with the value from (2) and solving for ε_{xd}

$$\varepsilon_{xd} = \frac{V_d}{V_u}(1+\varepsilon_{xu})-1 \qquad (3)$$

Equation (3) is a mathematical statement of the normal strain rule.

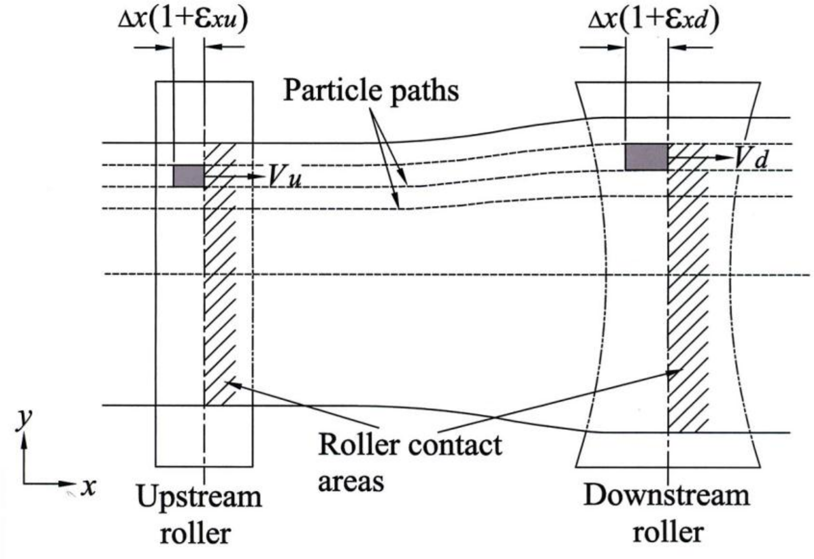

Figure 4
The normal strain rule

Area (a') shown in Figure 4 will not necessarily be rectangular. In cases, such as concave rollers, there will be shear stress at some points across the web at the roller entry. This does not, however, affect the validity of the normal strain rule. As illustrated in Figure 5, the shear strain will be equal to $\phi' - \phi$. Since ϕ' is the normal entry angle, it will be zero. Therefore, (a') becomes a parallelogram and its area will be the same as a rectangle of the same height and length. Furthermore, the length of the parallelogram, $(1+\varepsilon_x)dx$, will be the same as if the shear were not present.

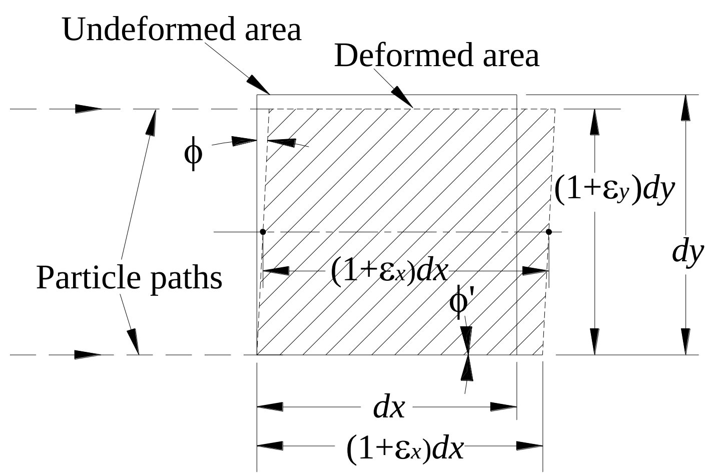

Figure 5
Effect of shear on the normal strain rule

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3 . Shelton, J. J., “Lateral Dynamics of a Moving Web,” Ph.D. Thesis, Oklahoma State University, Stillwater, Oklahoma, July 1968